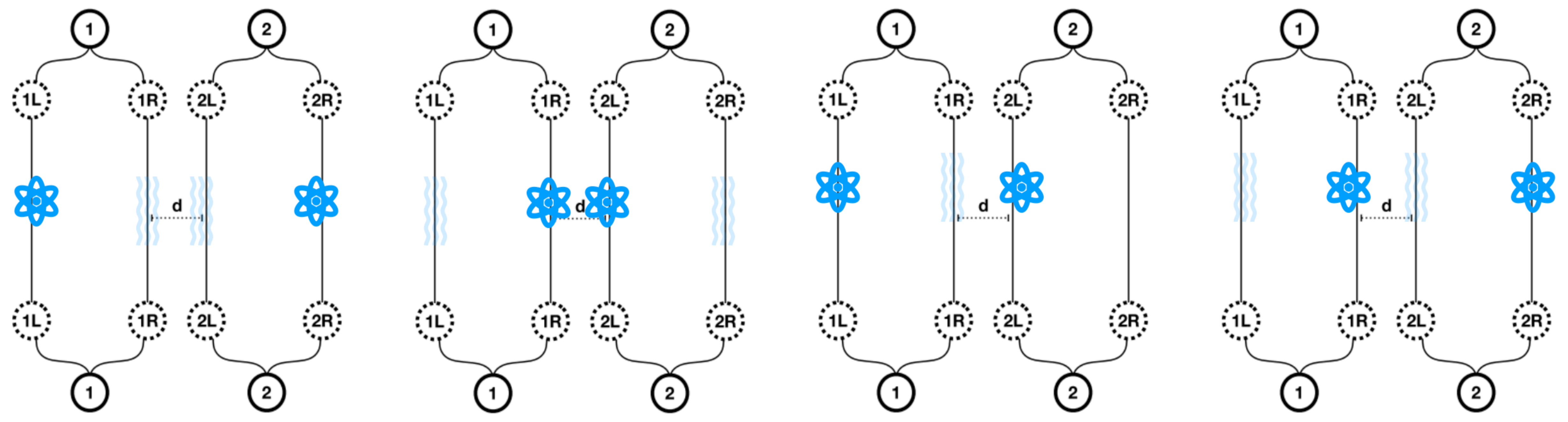

Bohmian trajectory gravity

in the BMV quantum gravity experiment

Thomas C Andersen: nSCIr, Canada, tandersen@nscir.ca

The recent experimental proposals by Bose et al. and Marletto et al. (BMV) outline a way to test for the quantum nature of gravity by measuring gravitationally induced phase accumulation over the paths of two entangled $\sim 10^{-14}kg$ masses. This work predicts the outcome of the BMV experiment in Bohmian trajectory gravity - where classical gravity is assumed to couple to the particle configuration in each Bohmian path. Bohmian trajectory gravity predicts that there will quantum entanglement. This is surprising as the gravitational field is treated classically.

The BMV experiment

In the BMV experimental proposal, Bose[1], and Marletto and Vedral[2] propose to measure the expected behaviour of quantum systems when particles interact only via mutual gravitational attraction. Christodoulou and Rovelli [3]:

> *...detecting the effect counts as evidence that the gravitational field can be in a superposition of two macroscopically distinct classical fields and since the gravitational field is the geometry of spacetime (measured by rods and clocks), the BMV effect counts as evidence that quantum superposition of different spacetime geometries is possible, can be achieved[3]*

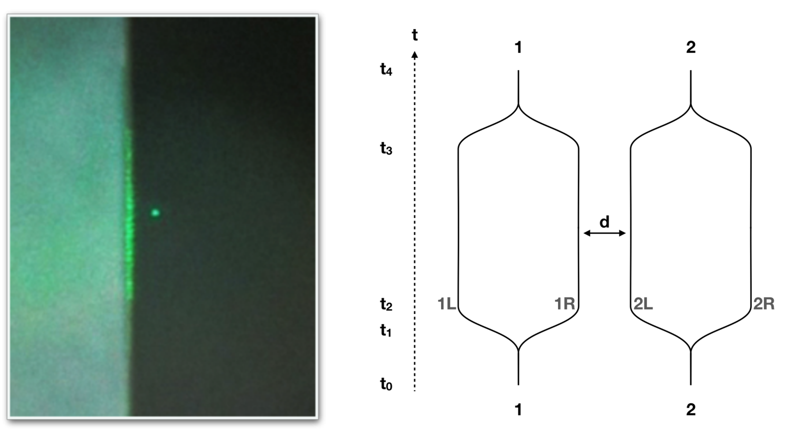

Figure 1: Particles 1 and 2 start off in the spin up $|u\rangle$ (defined as +z) direction, and are spatially split into superposed $|L\rangle + |R\rangle$ states by a Stern-Gerlach mechanism. The particles fly through the experiment at the same time, and then through a second Stern-Gerlach where they are brought back into up/down state.

Christodoulou and Rovelli[3] point out that the effect can be understood as the time dilation effects of one mass on the other. Although the time dilation effect is tiny, the experimental effect is amplified as the experimenters can take advantage of the extremely high Compton frequency of the massive (in the quantum particle sense) particle. The experimenters only need to measure the *difference* in phase accumulation along different paths, and once that phase difference is $\sim \pi$ the experimenters will detect that the state has become quantum entangled.

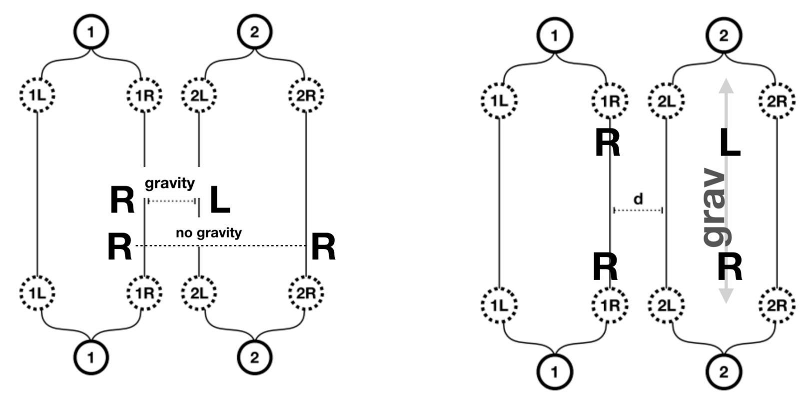

Figure 2: Left (entanglement): In quantum gravity particle 1 on the right branch feels gravity from both 2L and 2R in superposition. Right (no entanglement): In Rosenfeld style semi-classical gravity, particle 1R feels the same (negligible) gravity no matter which branch particle 2 is on.

Initial State

Each particle is passed through a typical Stern-Gerlach device, where L and R represent spins in the $\pm x$ direction.

$$|u_1\rangle = \frac{|L_1\rangle + |R_1\rangle}{\sqrt{2}} \quad , \quad |u_2\rangle = \frac{|L_2\rangle + |R_2\rangle}{\sqrt{2}} \tag{1}$$

and

$$u, d, \sigma_x, \sigma_y, \sigma_z \tag{2}$$

take on their usual meanings. The state at t_1 is thus:

$$|\Psi_{t_1}\rangle = \frac{1}{2}\Big(|LR\rangle + |RL\rangle + |LL\rangle + |RR\rangle\Big). \tag{3}$$

Measuring quantum entanglement

We wish to quantify the entanglement of the final state in the quantum gravity models considered here. A well chosen entanglement witness is a good measure. We use a witness similar to that of Bose et al.[1]. Witness:

$$\mathcal{W} = \langle \sigma_x^{(1)} \otimes \sigma_z^{(2)} \rangle + \langle \sigma_y^{(1)} \otimes \sigma_y^{(2)} \rangle \tag{4}$$

which shows entanglement (ρ is the density matrix)

$$\mathcal{W}(\rho) \leq 0. \tag{5}$$

See for instance[4]. The witness $\mathcal{W}$ is in the range [-2, 2]. Witness operators operate on density matrix representations of final states/mixtures.

For quantum gravity, if the experiment is arranged so that a phase change of π occurs on the R_1L_2 branch, we have

$$|\Psi_{t_4}\rangle_{qg} = \frac{1}{2}\Big(|LR\rangle - |RL\rangle + |LL\rangle + |RR\rangle\Big). \tag{6}$$

and our entanglement witness shows:

$$\mathcal{W}(\rho_{qg}) = Tr(\rho_{qg}W) = -2\ (quantum\ gravity). \tag{7}$$

For semi-classical gravity it's worthwhile to recall the physical model:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{8\pi G}{c^4}\langle\Psi|T_{\mu\nu}|\Psi\rangle \tag{8}$$

We see that Einstein's gravity connects to the *expectation* value of the matter-energy configuration T. The state at time t_4: $\Psi_{t_4} = \Psi_{t_0}$ (excepting overall phase)

and our entanglement witness shows:

$$\mathcal{W}(\rho_{sc}) = Tr(\rho_{sc}W) = 0\ (semi-classical\ gravity). \tag{9}$$

Bohmian trajectory prediction

de Broglie Bohm mechanics[5] uses non local hidden variables to create an interpretation of quantum mechanics where particles have well defined trajectories. Struyve writes:[6]

> *Bohmian mechanics solves the measurement problem by introducing an actual configuration (particle positions in the non-relativistic domain, particle positions or fields in the relativistic domain) that evolves under the influence of the wave function. According to this approach, instead of coupling classical gravity to the wave function, it is natural to couple it to the actual matter configuration.*

In equation form, comparing to the semi-classical form (8) above, we have:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{8\pi G}{c^4}T_{\mu\nu}(\varphi_B, g)\,. \tag{10}$$

Where φ_B is the trajectory of one member of the ensemble of Bohmian trajectories, and $T_{\mu\nu}(\varphi_B, g)$ is the stress energy tensor as determined by the specific trajectory in combination with perhaps other (e.g. earth's gravity) sources.

In order to interpret what happens in the BMV experiment assuming Bohmian trajectory gravity governs the quantum - gravity coupling consider:

Superposition: since gravity couples directly to each particle position, gravity is not in a superposition, instead the configuration of the gravitational field changes for each run of the experiment, with 4 possible configurations of particle trajectories.

Measurement: There is no measurement taking place, so there is no collapse. The effect on the quantum state when the particle trajectories take the 1R and 2L paths is for the wave function to be altered in the same way as in the quantized gravity solution.

The end result is that at t_4 there are two possible states, we have a mixed state represented with a density matrix, ρ_{bt}.

3/4 of the time, there will be no gravitational interaction and the final state will be the same as the starting state at t_1, while 1/4 of the time there is gravitational time dilation and the final state will be the same as that for quantized gravity.

$$75\% \quad |\Psi_{t_4}\rangle_{bt} = \frac{1}{2}\Big(|LR\rangle + |RL\rangle + |LL\rangle + |RR\rangle\Big) \tag{11}$$

$$25\% \quad |\Psi_{t_4}\rangle_{bt} = \frac{1}{2}\Big(|LR\rangle - |RL\rangle + |LL\rangle + |RR\rangle\Big) \tag{12}$$

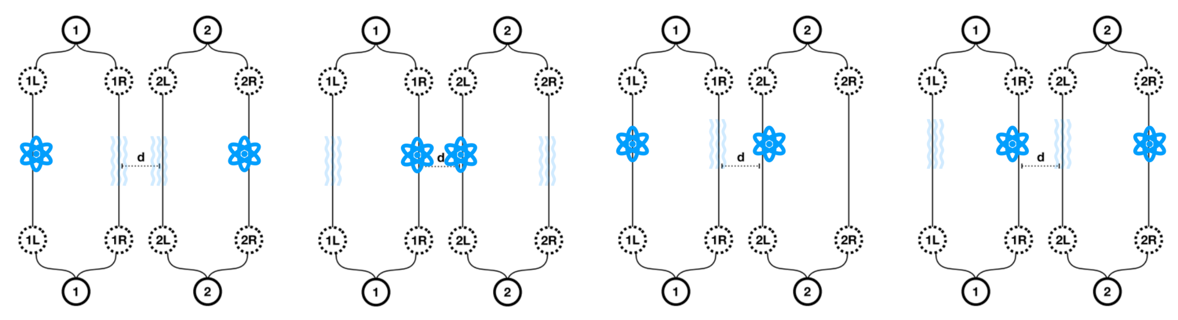

Figure 3: Four runs of the experiment showing Bohmian trajectories. Only in the second run from the left will there be gravitational interaction between the trajectories.

Evaluating the witness

$$\mathcal{W}(\rho_{bt}) = Tr(\rho_{bt}W) = -\frac{1}{2}\ (Bohmian\ trajectory). \tag{13}$$

The state is entangled. The witness shows a result 1/4 of that of the maximally entangled quantized gravity case.

Thus we have created entanglement with a classical single valued field.

Discussion

The BMV (and related) experiments provide a good probe of the nature of the interaction of gravity and quantum mechanics. The experiment proposed by BMV while very challenging to perform, is simple to study theoretically.

Of course some form of quantized gravity is the most popular prediction for the BMV experiment[3], with an entangled final state showing an entanglement witness $\mathcal{W} \sim -2$. The semi-classical model does not show any entanglement. The result prediction for Bohmian trajectory gravity shows entanglement with a witness of $\mathcal{W} \sim -\frac{1}{2}$.

Semi-classical gravity is perhaps more complicated than Bohmian trajectory gravity. In semi-classical gravity the gravitational field has to somehow integrate the entire position space of the wave function (a non local entity) in real time (via the Schrödinger - Newton equation), in order to continuously use the expectation value as a source for the gravitational field. In Bohmian mechanics, the gravitational field connects directly to an existing 'hidden' particle position, which is conceptually simpler.

Acknowledgements

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